APPS Review Key

1. $f(x) = 2x^3 + 3x^2 - 12x \quad [-1, 2]$

$f'(x) = 6x^2 + 6x - 12$

$0 = x^2 + x - 2 \quad (x+2)(x-1) = 0 \quad x = -2 \quad \boxed{x = 1}$

x	f(x)	
-1	13	max
2	4	
1	-7	min

2. ∞

$\frac{4x^3}{2x}$

$\frac{12x^2}{2}$

3. $\lim_{x \to 0} \frac{\sin 2x}{\sin 5x}$

$\lim_{x \to 0} \frac{2\cos 2x}{5\cos 5x}$

$\boxed{2/5}$

4. $\boxed{2}$

5. $\lim_{x \to 1} \frac{\ln x^2}{x^2 - 1} \quad \frac{2\ln x}{x^2 - 1}$

$\lim_{x \to 1} \frac{2/x}{2x} = \frac{2}{2} = \boxed{1}$

6. $f(x) = 2x^3 - 6x + 3$

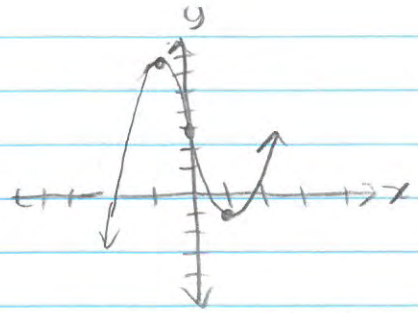
$f(0) = 3$ no asymp.

$f'(x) = 6x^2 - 6 \quad f''(x) = 12x$

$0 = 6(x^2 - 1) \quad x = 0$

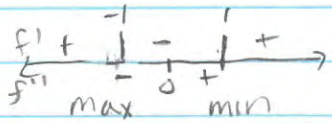
$x = \pm 1$

x	f(x)
-1	7
0	3
1	-1



crit. value

$x = \pm 1$



inc: $(-\infty, -1) \cup (1, \infty)$ $f' > 0$

dec: $(-1, 1)$ $f' < 0$

min: $x = 1$ f' changes from − to +

max $x = -1$ f' changes from + to −

POI $x = 0$ f'' changes sign

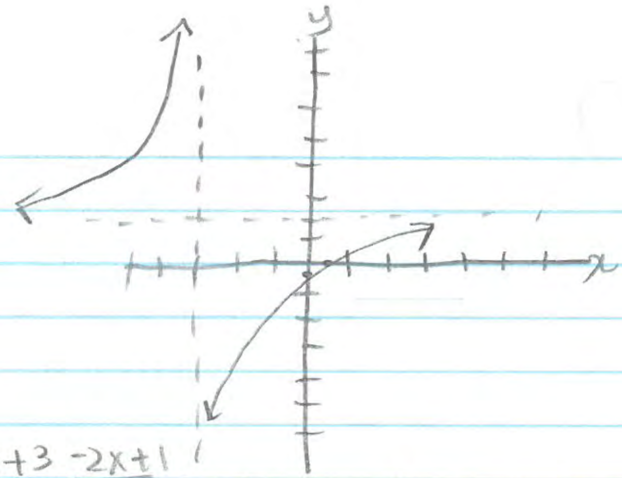
cup: $(0, \infty)$ $f'' > 0$

cdown: $(-\infty, 0)$ $f'' < 0$

7. $f(x) = \dfrac{2x-1}{x+3}$ $(0, -1/3)$ $(1/2, 0)$

$x = -3$ VA

$y = 2$ HA



$f'(x) = \dfrac{(x+3)(2) - (2x-1)(1)}{(x+3)^2} = \dfrac{2x+3-2x+1}{(x+3)^2} = \dfrac{4}{(x+3)^2}$

f' + | + (–3); + –3 –

crit. value: $x = -3$

$f'>0$ inc: $(-\infty, -3) \cup (-3, \infty)$

$f''<0$ CD: $(-3, \infty)$

$f''>0$ CU: $(-\infty, 3)$

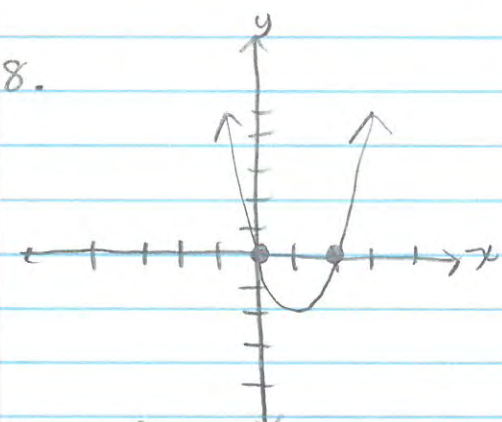
no poi or extrema

$f''(x) = 4(x+3)^{-3}(-2)'$

$= -8(x+3)^{-3}$

$x = -3$

8.



(0,0)
(2,0)

dec until x=1
incr. after x=1
min at x=1

c up

9. $x^3 + xy + y^4 = 19$

$3x^2 + x\frac{dy}{dx} + y + 4y^3\frac{dy}{dx} = 0$

$(x + 4y^3)\,dy/dx = -y - 3x^2$

$\dfrac{dy}{dx} = \dfrac{-y - 3x^2}{x + 4y^3}$

at (1, 2)

$\dfrac{dy}{dx} = \dfrac{-2-3}{1+4(2)^3} = \dfrac{-5}{33}$

$y - 2 = \dfrac{-5}{33}(x-1)$

f(1.1)

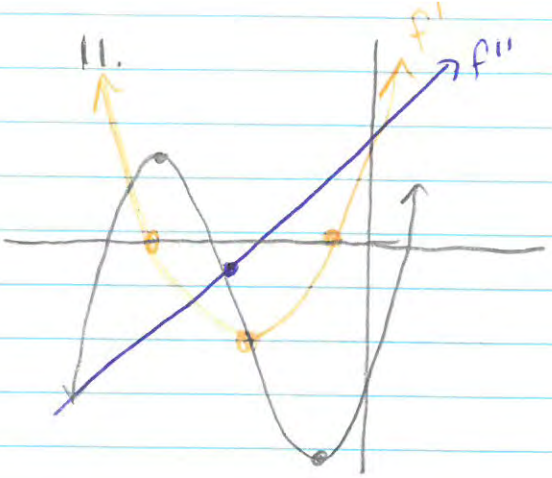
$y - 2 = -5/33\,(1.1-1)$

$y - 2 = -5/33\,(.1)$

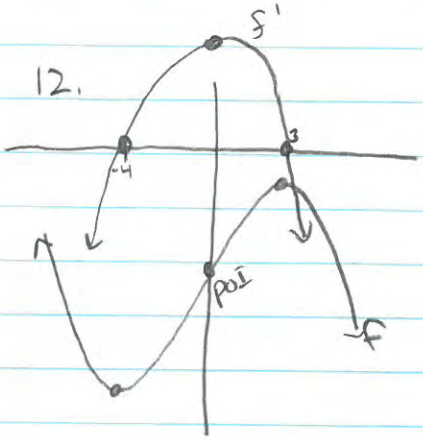
$y \approx 1.985$

10. abs max none
abs min $(-1,5)$

11.



12.



a) $(-4,3)$ f is increasing
b) $(-\infty,-4) \cup (3,\infty)$ f is decreasing
c) $(0,\infty)$ f conc. down
d) $(-\infty,0)$ f cup

13. $f(x) = 4x^3 + ax^2 + bx + K$

$f'(x) = 12x^2 + 2ax + b$

$0 = 12(-1)^2 + 2a(-1) + b$

$0 = 12 - 2a + b$

$0 = 12 - 2(24) + b$

$0 = 12 - 48 + b$

$\boxed{b = 36}$

$x = -1$ local min $f' = 0$

$x = -2$ POI $f'' = 0$

$f''(x) = 24x + 2a$

$0 = 24(-2) + 2a$

$48 = 2a$ $\boxed{a = 24}$

$7 = 4(2)^3 + 24(2)^2 + 36(2) + K$

$7 = 32 + 96 + 72 + K$

$\boxed{-193 = K}$

14. $t \geq 0$ $v(t) = t \sin(t^2)$

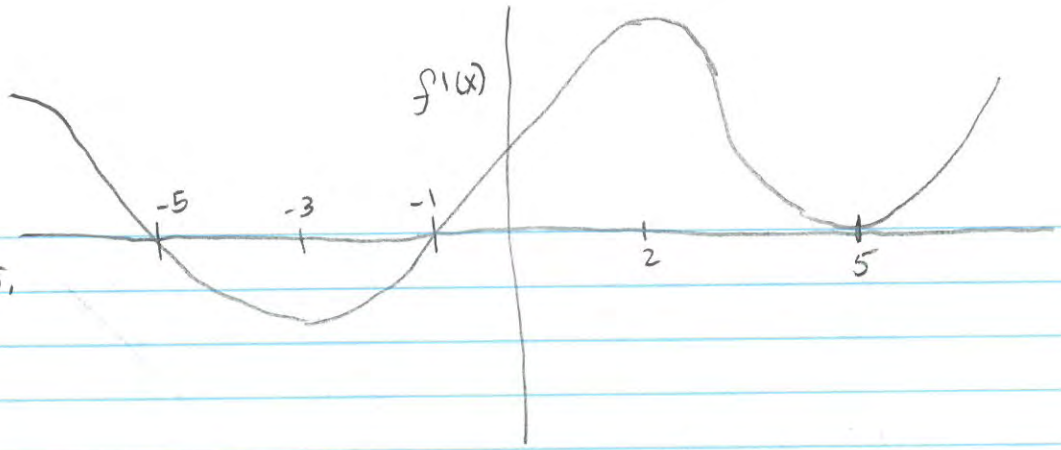
a) $v(1.5) = 1.5 \sin(1.5)^2 = 1.167$

particle is moving up because $v(1.5) > 0$

b) $v' = t(\cos t^2)(2t) + \sin t^2$

$v'(1.5) = 1.5 \cos 2.25 \cdot 3 + \sin 2.25 \approx -2.049$

v is decreasing since $v' < 0$, speed is also decreasing since $v > 0$ $v' < 0$



15.

a) f has a rel min when f' changes from negative to positive at $x = -1$

b) f has a rel max when f' changes from positive to negative at $x = -5$

c) f'' is slope of f' $f'' < 0$ for $(-7,-3) \cup (2,5)$

16. $f(x) = x^4 + 4x^2 + 1$ $[-3,3]$

a) cont. ✓ diff ✓

$f(-3) = 81 + 4(9) + 1 = 118$ ✓

$f(3) = 81 + 4(9) + 1 = 118$

b) $f'(x) = 4x^3 + 8x$

$0 = 4x(x^2 + 2)$

$\boxed{0 = x}$ $x^2 \neq -2$

c)

-3	118
3	118

max's

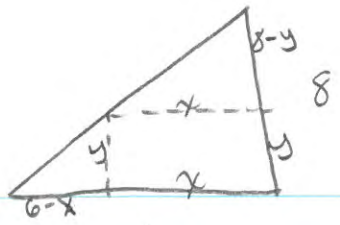
0	1

min

17) av. rate $\frac{-56-44}{12} = \frac{-100}{12} \approx -8.3°F/hr$

By the MVT the rate of cooling was $-8.3°F/hr$ at least once in the 12 hr. interval.

18.



max area

$A = xy$

$\frac{y}{6-x} = \frac{8-y}{x}$

$xy = 48 - 8x - 6y + xy$

$6y = 48 - 8x$

$y = 6 - 4/3x$

$A = x(6 - 4/3x)$

$A = 6x - 4/3x^2$

$A' = 6 - 8/3x$

$0 = 6 - 8/3x$

$6 = 8/3x$

$\frac{9}{4} = \frac{18}{8} = x$

+ | −

9/4 max

if $x = 9/4$ $y = 6 - 4/3(9/4) = 3$

dimensions (9/4, 3)

19.

$V = 32$ $V = \pi r^2 h$ $32 = \pi r^2 h$

$\frac{32}{\pi r^2} = h$

SA:

$A = 2\pi rh + \pi r^2$

$A = \frac{2\pi(r)(32)}{\pi r^2} + \pi r^2 = \frac{64}{r} + \pi r^2$

$A' = -64r^{-2} + 2\pi r$

$0 = \frac{-64}{r^2} + 2\pi r$ $r^{-2}(-64 + 2\pi r^3)$

$\frac{64}{2\pi} = r^3$

$\frac{32}{\pi} = r^3$

$r \approx \sqrt[3]{32/\pi}$

$r \approx 2.168$

− | +

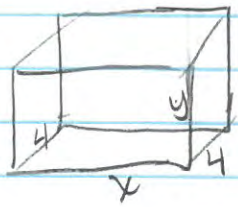
min

2.168

$r \approx 2.168$

$h \approx 2.167$

20.



$V = 36$ minimize cost.

$36 = 4xy$

$\frac{9}{x} = y$

cost: $C = 4x(10) + (4y)(2)(5) + 2xy(5)$

$C = 40x + 40y + 10xy$ $\quad 10x(\frac{9}{x})$

$C = 40x + \frac{360}{x} + 90$

$C' = 40 + 360x^{-2}$

$0 = 40 - 360x^{-2}$

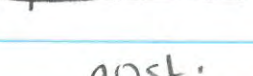
$40 = \frac{360}{x^2}$ $\quad x^2 = 9$

$x = \pm 3$

$\boxed{x = 3}$

$\boxed{y = 9/3 = 3}$

cost: $C = 120 + 120 + 90 = \$330$



mc

1. B	$x=0$ discon. $x=2$ vert. tan.
2. D	↗
3. B	$y' = 3x^2 + 6x$ $\quad y'' = 6x+6$ $\quad 6x = -6$ $\quad x = -1$ POI at $x=-1$ $\quad y' = 3-6 = -3$ $y = (-1)^3 + 3(-1)^2 + 2 = 4$ $\quad$ Find $\frac{dy}{dx}$ $y - 4 = -3(x+1)$ $\quad$ Find y $y = -3x + 1$
4. E	$y' = -5(x-2)^{-1}$ $\quad y' = -5(x-2)^{-2}$ $\quad y'' = 10(x-2)^{-3}$ y'' + | − $\quad x=2$ 2
5. B	decreasing / cdown
6. D	I: yes! ext. occur at crit. values II no when f" changes sign III: yes! $\quad$ note: all crit values are not extrema!

mc continued

7\. C — $f' = 4Kx - 5K$ $\quad f'' = 4K$

$4K < 0$

$K < 0$

8\. D — $f(x) = x^3$ $\quad m = \frac{8-(-1)}{2-(-1)} = \frac{9}{3} = 3$ $\quad f' = 3x^2$

$f(-1) = -1$ $\quad 3x^2 = 3$

$f(2) = 8$ $\quad x^2 = \pm 1$

Both in interval

9\. B — $g(x) < 0$ $\quad x = \pm 2$ $\quad f'$: $-$ (∪) -2 $+$ (∩) 2 $-$

10\. C — $f'' = x(x+1)(x-2)^2$

$x = 0 \quad x = -1 \quad x = 2$

$+$ (−1) $-$ (0) $+$ 2 $+$

11\. C — cup, dec. and $f(3) = 0$

$f'' > 0$ $\quad f' < 0$

$+$ $\quad -$ $\quad f' < f < f''$

12\. D — $y' = x^2 + 10x$ $\quad y'' = 2x + 10$ $\quad -$ -5 $+$

$-10 = 2x$

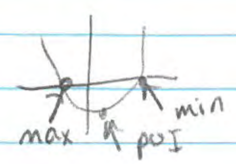
$-5 = x$

13\. C — $f' = 3x^2 + 12$ $\quad$ f is always increasing since f' is always pos.

$3x^2 = -12$

$x^2 = -4$

14\. B — needs max left / min right



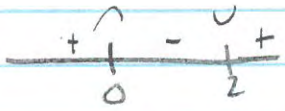
15\. D — $\frac{4n^2}{n^2}$

16. D

16. $f' = 3x^2 - 6x$

$0 = 3x(x-2)$

$x=0 \quad x=2$



x	f
-2	-8
0	12
4	(28)

$f = x^3 - 3x^2 + 12$

$-8 - 12 + 12$

$64 - 48 + 12$

occurs at $x=4$

17. E

$$\frac{2x+b}{x+3}$$

18. C

$y' = -2x^{-3} + 3x^{-4}$

$y'' = 6x^{-4} - 12x^{-5}$

$0 = 6x^{-4} - 12x^{-5}$

$0 = 6x^{-5}(x-2)$

$x \neq 0 \quad x = 2$

$y = x^{-2} - x^{-3} \qquad x \neq 0$